H2 Gas-Filled Helical FOFO Snake for Initial 6D Ionization Cooling of Muons

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**Abstract.** An H2 gas-filled channel for 6D ionization cooling of muons is described which consists of periodically inclined solenoids of alternating polarity with 325MHz RF cavities inside them. To provide sufficient longitudinal cooling LiH wedge absorbers are placed at the minima of transverse beta-function between the solenoids. An important feature of such channel (called Helical FOFO snake) is that it can cool simultaneously muons of both signs. Theoretical considerations as well as results of simulations with G4beamline are presented.

Keywords: muon beam, ionization cooling, beam dynamics.

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**1. Lattice**

One period of the channel is shown in Fig. 1 (top). It consists of:

•  Alternating solenoids (coil parameters: *L*= 30 cm, *Rin*= 42 cm, *Rout*= 60 cm) placed with step 70 cm along the axis. With current density 94.6 A/mm2 the solenoids provide focusing with betatron phase advance ≈74°/step for muon momentum 230 MeV/c. To create transverse magnetic field component (Fig. 1 second plot from the top) the solenoids are periodically inclined in rotating planes *x*⋅cos(*φk*)+*y*⋅sin(*φk*) = 0, *φk*= *π*(1–2/*Ns*)(*k* + 1), *k*=1,2,…, *Ns*, *Ns* being the number of solenoids/period. In the considered case *Ns* = 6 and the rotation angles *φk* are: 4*π*/3, 0, 2*π*/3, 4*π*/3, 0, 2*π*/3; *φ* = 0 corresponds to pitching in the vertical plane. The chosen pitch angle of 2.5 mrad is too small to be visible in the picture.

•  Paired RF cavities (*fRF*= 325 MHz, *L*= 2×25 cm, *Emax*= 25 MV/m) filled with H2 gas with density 20% that of liquid hydrogen. Radius and thickness of Be windows are reduced in 3 steps along the channel: *Rw*= 30 cm, *w* = 0.12 mm (first 10 periods), *Rw*= 25 cm, *w* = 0.10 mm (next 10 periods) and *Rw*= 20 cm, *w* = 0.07 mm (last 10 periods).

•  LiH wedge absorbers providing additional longitudinal cooling. Although the momentum compaction factor of the lattice is positive (equilibrium orbit and dispersion have the same sign and shape as can be seen in two lower plots in Fig.1) it is not sufficient for longitudinal cooling so the wedges had to be added. The wedge angle smoothly varies along the channel from 0.17 rad to 0.20 rad. The tip of the wedge just intercepts the channel axis so the muons on the equilibrium orbit (Fig. 1 third plot from the top) traverse no more than 0.3 mm of LiH.

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| Figure 1. Layout of one period of the HFOFO lattice (top), magnetic field for muon momentum 230 MeV/c (second from top), μ+ equilibrium orbit and dispersion (third from top and bottom). |

An important feature of the design is that μ− in solenoids 4, 5, 6 see exactly the same forces as μ+ in solenoids 1, 2, 3 and vice versa, so that μ− and μ+ orbits have exactly the same form with longitudinal shift by half period (three solenoids) but are not mirror-symmetric as one might expect. This allows us to find such orientation of wedge absorbers (with periodicity = 2) that they provide longitudinal cooling for both μ− and μ+ at the same time.

Table 1. The normal mode tunes and normalized equilibrium emittances

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| --- | --- | --- | --- |
| Mode | I | II | III |
| Tune | 1.2271 + 0.0100 i | 1.2375 + 0.0036 i | 0.1886 + 0.0049 i |
| Emittance (mm) | 2.28 | 6.13 | 1.93 |

The transverse normal modes (I and II) cooling rates and equilibrium emittances can be equalized with the help of a unipolar quadrupole field (not necessarily of constant gradient). Such field works for both μ− and μ+ despite breaking the translational symmetry between the two beams. However, a strong β-beat is excited (Fig. 2) increasing slightly the 4D emittance.

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| Figure 2. μ+ transverse β-functions with no (top) and with constant quadrupole field (bottom) of indicated strength. |

In the present version of the channel the quadrupole field is turned off.

**2. β⊥****matching to D. Neuffer’s rotator**

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| Figure 3. Magnetic field in the transition area (left) and β-function for constant momentum (right) |

D. Neuffer’s frontend has homogeneous 2 T magnetic field, the corresponding value β⊥= 77cm (p = 230MeV/c) is significantly larger than that in the FOFO channel. To achieve matching (in absence of solenoid inclination) the current in the first 4 FOFO solenoids was adjusted. Figure 3 presents the resultant magnetic field and β-function obtained for constant values of momentum by solving differential equation

 (1)

where , , prime means differentiation by *z*.

A significant difficulty associated with rapid β-function reduction is increase in the equilibrium momentum for muons with large transverse amplitudes: due to longer path these particles must move faster to be in synchronism with RF.

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| Figure 4. Total momentum of the reference particle in the beginning of the channel (left) and solution of Eq.(1) with such momentum dependence (right). |

To leave the average momentum ~ constant the reference momentum value should be reduced, but not too fast to avoid excitation of longitudinal oscillations. Figure 4 shows the reference particle total momentum in the end of the rotator solenoid (which ends at *z* = -35 cm) and in the beginning of the FOFO channel. A sharp drop at *z* = 40 cm is caused by 4 mm thick stainless steel wall containing the high pressure hydrogen (assumed to be at 77°K). By mistake the hydrogen was introduced to the left of the wall as well, the associated energy loss was compensated with RF cavities.

The relatively slow decline in momentum after the pressure wall was designed to counteract the above-mentioned effect of β-function reduction. Solution of Eq.(1) with momentum dependence presented in Fig.4 (left) shows that β-matching is well-preserved.

**3. Momentum and orbit matching**

The HFOFO snake has a peculiar limitation on the momentum from above associated with change in sign of the slippage factor [1]. In the result the drop in the reference momentum shown in Fig. 4 (left) is not sufficient to ensure good transmission through the channel. Further (adiabatic) momentum reduction is achieved by decreasing the magnetic field and adjusting phase of RF cavities. At the same time the helical orbits are introduced.

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| Figure 5. Total momentum of the μ+ reference particle in the first part of the channel (left) and its transverse orbit (right) |

To place both μ+ and μ− reference particle on the orbits in a periodic channel inclination of solenoids 3-9 was used (the total of 14 variables to satisfy just 8 conditions). Matching of dispersion functions has not been attempted.

**4. Cooling muon beam from D. Neuffer’s rotator**

The cooling was simulated with the muon beam obtained with D. Neuffer’s original frontend design which does not include the chicane. Parameters of the initial muon distribution obtained with Gaussian fit [2] which automatically suppresses contribution from the tails[[1]](#footnote-1) are presented in Table 2.

Table 2. Parameters of muon beams from the front end

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **N**  **total** | **N 150<p<360** | **N**  **core** | **p(central) MeV/c** | **σp MeV/c** | **σ⊥ cm** | **εmN cm** | | | **ε6D cm3** |
| μ+ | 11755 | 7998 | 7329 | 248.0 | 29.8 | 7.6 | 1.2 | 2.2 | 2.4 | 6.2 |
| μ− | 12396 | 9020 | 8248 | 248.8 | 28.2 | 7.4 | 1.2 | 2.1 | 2.2 | 5.6 |

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| Figure 6. Design momentum and central momentum of μ+ and μ− beams obtained with Gaussian fit [2] |

Due to the already mentioned effect of large transverse amplitudes on the equilibrium momentum the RF timing should be readjusted to work with beams from the front end. Figure 6 shows central momentum dependence obtained with Gaussian fit over the full length of the channel. Evolution of the normal mode emittances and beam core intensity obtained by tracking with G4beamline [3] is shown in Fig. 7, solid lines correspond to μ+ while dashed lines give values for μ−.

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| Figure 7. Normal mode emittances (left) and beam core intensity (right) for μ+ (solid lines) and μ− (dashed lines) |

**5. Exit out of the HFOFO channel**

For use in a muon collider the μ+ and μ− beams should be separated, then each beam should be merged in a single bunch and cooled further. Since the separator employs a constant magnetic field the HFOFO channel must be matched to it. This was done in a similar manner as matching to the rotator solenoid. The separator field was also chosen to have strength 2 T, so a direct comparison with the initial phase space distribution can be made (Fig. 8).

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| Figure 8. Phase space distribution of the initial μ+ beam (blue) and of the cooled beam in the exit solenoid (red). All bunches were projected onto the same RF bucket in the right plot. No cuts applied. |

Reduction in the momentum distribution width is illustrated by histogram in Fig. 9.

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| Figure 9. Total mechanical momentum distribution in the initial μ+ beam (blue) and in the cooled beam in the exit solenoid (pink). Distribution in μ− beams looks similar. |

Table 3. Parameters of the cooled muon beams in the exit solenoid (Gaussian fit)

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **N**  **total** | **N 150<p<360** | **N**  **core** | **p(central) MeV/c** | **σp MeV/c** | **σ⊥ cm** | **εmN mm** | | | **ε6D mm3** |
| μ+ | 5378 | 5167 | 5010 | 208.2 | 16.1 | 3.3 | 1.9 | 3.6 | 7.6 | 51. |
| μ− | 5896 | 5743 | 5499 | 207.7 | 15.5 | 3.4 | 1.6 | 4.6 | 7.2 | 51. |

Transmission – if all particles are counted – is only 46% for μ+ and 48% for μ− (decays included) due to very long tails of high-energy muons in the initial distribution. However, if only particles in the Gaussian core are counted, the transmission improves to 68% for μ+ and 67% for μ−.

**6. Outlook**

The major difficulty with the present HFOFO design is a smaller value of the transverse β-function compared to that in the rotator solenoid. This leads to two undesirable effects: increase in the beam angular spread (up to total reflection of some particles!) and additional energy spread due path lengthening for particles with large transverse amplitudes.

A version of the HFOFO was designed with three RF cavities per solenoid in order to increase the period length and consequently the β-function. Using this lattice as the first stage it will be possible to increase beam intensity by ~5%.

Also, one can append a stage with shorter RF cavities (say, 15 cm in length) and smaller gaps between them since the required LiH wedge height for cooled beams is smaller as well, and reduce the period length and consequently the β-function and equilibrium emittances by ~ one third.

**7. References**

[1] Y. Alexahin, Helical FOFO snake for 6D ionization cooling of muons, AIP Conf.Proc. 1222 (2010) 313-318.

[2] Y. Alexahin, Computing Eigen-Emittances from Tracking Data, MAP-doc-4358, FNAL 2013.

[3] T. Roberts, http://g4beamline.muonsinc.com

**Appendix**

The G4BL deck is also submitted together with this note and consists of a master file track\_v7.in which describes all used elements and calls in a number of auxiliary files:

•  initial.dat, a user provided file with particle data, the user must also provide parameter $beamtime value which is the average time of μ+ in initial.dat modulo RF period *TRF*. If there is no data for μ+ then μ− time should be taken and *TRF*/2 added.

•  solangles1.dat and solangles2.dat which define solenoid inclinations required to open and close the helix

•  detectors.txt

•  abs\_place7\_31.txt, RFplace7\_31.txt and sol\_place7\_31.txt which place the elements and provide particular values for the LiH absorber wedge angles, RF phases and solenoid currents which are varying along the channel.

1. For comparison the r.m.s. emittances of the μ+ normal modes after 150 < p (MeV/c) < 360 cut are: 1.2, 2.1, 5.0 (cm) [↑](#footnote-ref-1)